AN204469

# FM3 Family 3-Phase PMSM FOC Control

**Associated Part Family: FM0+ / FM3 / FM4 Series**

This application note describes the FOC control of a 3-phase PMSM which includes, the structure of a 3-phase PMSM and motor driving principle, FOC control system, Core modules and Mathematical model of a 3-phase PMSM.

## Contents



# 1 Introduction

## 1.1 Purpose

This document describes the FOC control of a 3-phase PMSM. The following contents are included:

- The structure of a 3-phase PMSM and motor driving principle
- FOC control system
- Core modules
- Mathematical model of a 3-phase PMSM

## 1.2 Definitions, Acronyms and Abbreviations

| | |
|---|---|
| PMSM | Permanent Magnetic Synchronous Motor |
| SVPWM | Space Vector Pulse Width Modulation |
| FOC | Field Oriented Control |

## 1.3 Document Overview

The rest of document is organized as the following:

Section 1 explains Introduction.

Section 2 explains Structure of a 3-Phase PMSM and Motor Driving Principle.

Section 3 explains FOC Control.

Section 4 explains Core Modules.

Section 5 explains Mathematical Model of a 3-Phase PMSM.

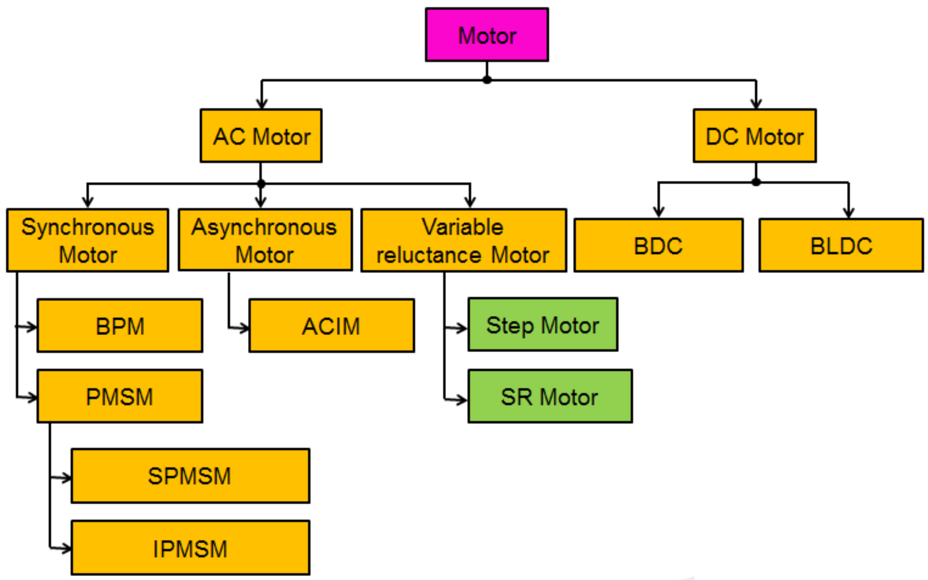
Section 6 explains Additional Information.

# 2 Structure of a 3-Phase PMSM and Motor Driving Principle

## 2.1 Motor Category

Synchronous motors can be divided into several different types. The figure below shows a simple classification tree of electric motors. In this document, the FOC control of 3-phase PMSM is to be introduced, which is highlighted with the green color in below figure.

Figure 1. Motor Category



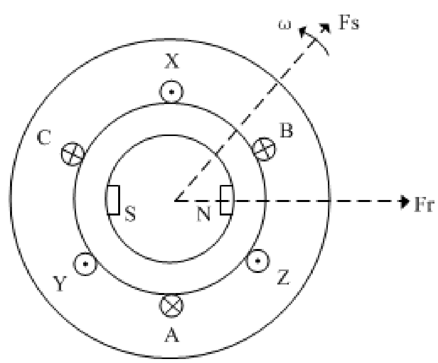

## 2.2 Structure of a 3-Phase PMSM

A 3-phase PMSM is mainly composed of two parts: the stator and the rotor.

At stator side, the 3-phase windings are coiled on the stator core. The windings of 3 phases are separately placed by the rule of 120 degrees angle to generate a round rotating magnetic field (Fs) when a 3-phase AC current goes through the 3-phase windings. The separated 3-phase winding placed by the rule of 120 degrees angle is named as 3-phase symmetric winding.

At rotor side, one or more pairs of permanent magnetic poles are mounted to offer a constant rotor magnetic field (Fr).

Figure 2. Structure of a 3-phase PMSM



Because Fs is a rotating magnetic field, the Fr will be dragged and follow the Fs. If the Fr cannot catch up with Fs, the rotor will rotate continuously. If the 3-phase current in 3-phase windings disappears, the Fs will disappear at the same time, and the rotor will stop.

## 2.3 Driving principle of 3-Phase PMSM

To keep rotor rotating, a continuous rotating stator magnetic field is necessary. Assume the 3-phase AC current can be expressed as:

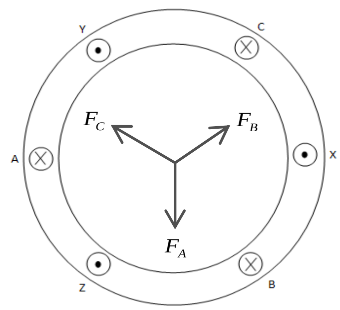
$$i_{AX} = I_m \sin(\omega t) \ldots (2.3-1)$$

$$i_{BY} = I_m \sin(\omega t - 120^\circ) \ldots (2.3-2)$$

$$i_{cZ} = I_m \sin(\omega t - 240^\circ) \ldots (2.3-3)$$

The 3-phase AC current goes through stator 3-phase winding, so three magnetic fields are created. By the formula of F= $Ni$ , the three magnetic fields can be expressed as:

Figure 3. Stator Magnetic Field of A 3-phase PMSM




$$F_A = F_{AX} = NI_m \sin(\omega t) \ldots (2.3-4)$$

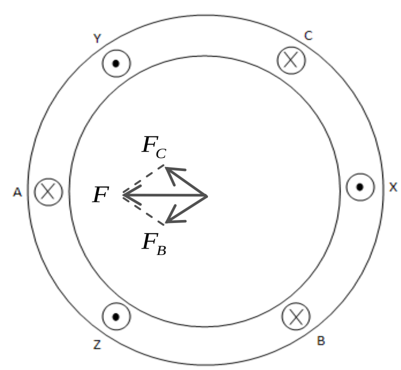
$$F_B = F_{BY} = NI_m \sin(\omega t - 120^\circ) \ldots (2.3-5)$$

$$F_C = F_{cZ} = NI_m \sin(\omega t - 240^\circ) \ldots (2.3-6)$$

By observing the synthesis magnetic F in different timing modes, which is combined by $F_A, F_B, and\ F_c$, it is easy to understand the rotating principle of stator magnetic field.

1. $\omega t = 0^\circ$

Figure 4. Stator Magnetic Field of a 3-Phase PMSM When $\omega t = 0^\circ$



$$F_A = F_{AX} = 0 \ldots (2.3-7)$$

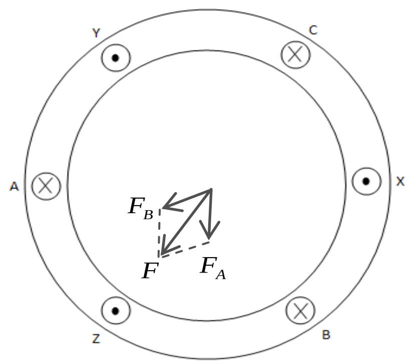
$$F_B = F_{BY} = -\frac{\sqrt{3}}{2} N I_m \ldots (2.3-8)$$

$$F_C = F_{cZ} = \frac{\sqrt{3}}{2} N I_m \ldots (2.3-9)$$

$$F = \frac{3}{2} N I_m, (the\ direction\ is\ shown\ in\ figure\ above) \ldots (2.3-10)$$

2. $\omega t = 60^\circ$

Figure 5. Stator Magnetic Field of a 3-Phase PMSM When $\omega t = 60^\circ$




$$F_A = F_{AX} = \frac{\sqrt{3}}{2} N I_m \ldots (2.3-11)$$

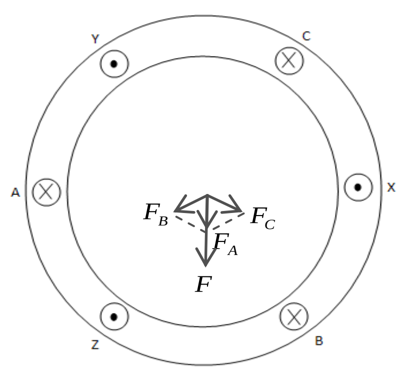
$$F_B = F_{BY} = -\frac{\sqrt{3}}{2} N I_m \ldots (2.3-12)$$

$$F_C = F_{cZ} = 0 \ldots (2.3-13)$$

$$F = \frac{3}{2} N I_m, (the\ direction\ is\ shown\ in\ figure\ above) \ldots (2.3-14)$$

3. $\omega t = 90^\circ$

Figure 6. Stator Magnetic Field of a 3-Phase PMSM When $\omega t = 90^\circ$



$$F_A = F_{AX} = NI_m \ldots (2.3 - 15)$$

$$F_B = F_{BY} = -\frac{1}{2}NI_m \ldots (2.3 - 16)$$

$$F_C = F_{cZ} = -\frac{1}{2}NI_m \ldots (2.3 - 17)$$

$$F = \frac{3}{2}NI_m, (the\ direction\ is\ shown\ in\ figure\ above) \ldots (2.3 - 18)$$

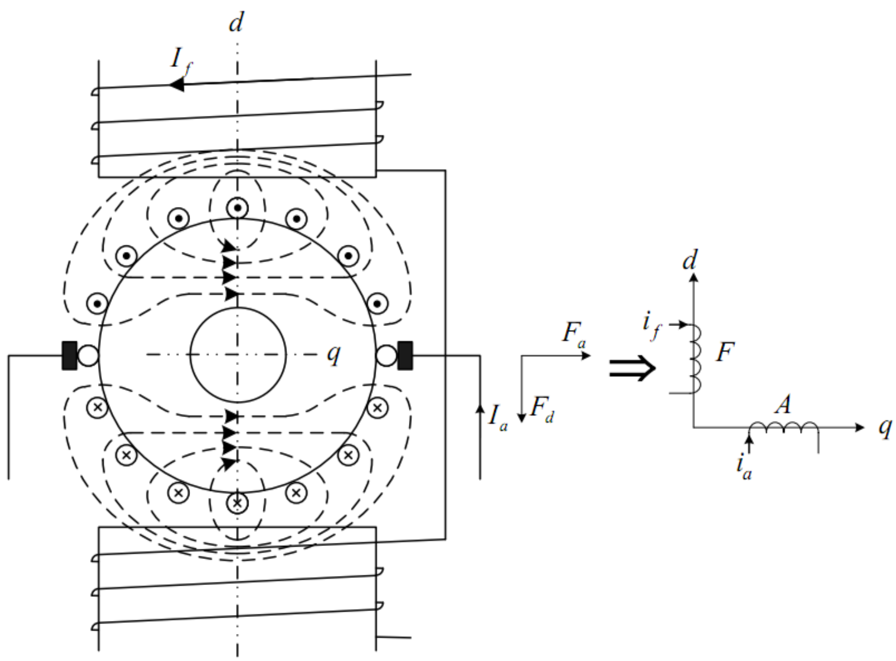
From the synthetic magnetic field F in different timing modes, it is obviously known that F is a rotating vector and furthermore the magnitude of F is a constant($\frac{3}{2}NI_m$). Now we can get a conclusion that a round rotating magnetic field is created if a 3-phase AC current goes through a 3-phase symmetric winding.

# 3 FOC Control

### 3.1.1 FOC Principle

Brush DC motor is the conventional DC motor with a long history. A big advantage of the brush DC motor is that its torque control and magnetizing control are decoupled, which makes brush DC motor easy to control. The brush DC motor decoupled control is shown in below figure.

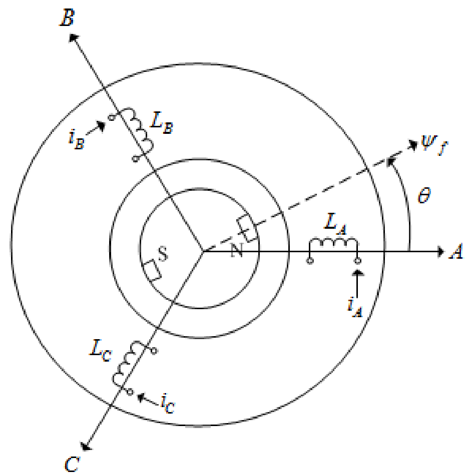
Figure 7. Brush DC Motor Decoupled Control




The magnetizing is controlled by magnetizing current ($I_f$), and the torque is controlled by torque current ($I_a$). The direction of the magnetizing magnetic field is parallel with d-axis (vertical direction), and the direction of the torque magnetic field is parallel with q-axis (horizontal direction). So these two magnetic fields do not influence each other. That is to say, it is decoupled between the 2 magnetic fields and motor’s magnetizing and torque can be adjusted individually. For example, the torque control formula is $T_e = C_m \emptyset I_a$, which means torque is only controlled by torque current $I_a$.

The condition of PMSM motor control is much more complex than a brush DC motor. The magnetic field of a 3-phase symmetry winding is a coupled magnetic field. We can discover the complex coupled relationship from the torque control formula.

Figure 8. The Coupled Magnetic Flux of A PMSM




$$T_e = \frac{1}{2} n_p [I_{ABC}]^T \frac{\partial [L_{ABC}]}{\partial \theta} [I_{ABC}] \ldots (3.1.1-1)$$

$$[L_{ABC}] = \begin{bmatrix} L_A & M_{AB} & M_{AC} \\ M_{BA} & L_B & M_{BC} \\ M_{CA} & M_{CB} & L_C \end{bmatrix} (M\ is\ mutual\ inductance),\ [I_{ABC}] = \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

From the expression of Te, it is easy to understand that the torque is determined by all 3-phase inductances (including self-inductance and mutual-inductance) and currents. Obviously, the torque control seems much more complex than a brush DC motor.

Coordinate transformation is just the way to simplify the PMSM torque control. By coordinate transformation, a PMSM control model is converted from A-B-C coordinate to d-q coordinate. The torque control formula is also converted into d-q coordinate, the formula is:

$$T_e = \frac{3}{2} n_p \psi_d I_q \ldots (3.1.1-2)$$

The simple formula in d-q coordinate makes the PMSM torque control as easy as a brush DC motor.

### 3.1.2 FOC Control Structure

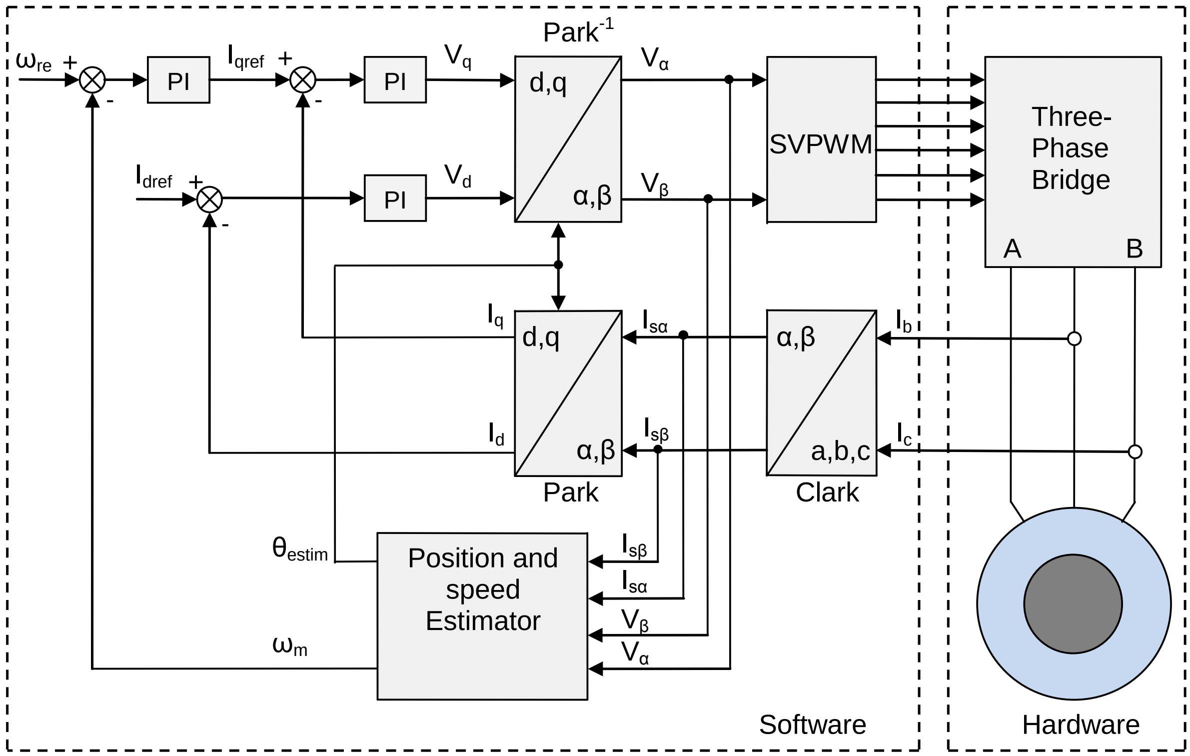
From the description above, the FOC core thinking is to make the torque control of PMSM as easy as a DC brush motor by a motor rotor magnetic field orientation technology. In the technology, the coordinate transformation method turns the motor module from the u-v-w coordinate to the rotational d-q coordinate, and the d-q coordinate rotational speed is the same as the stator magnetic field rotational speed. Then the control of a PMSM is simplified and the control performance is almost same as a DC brush motor.

Some PID regulators are added to adjust the motor output following the given input. By setting different PID parameters, system gets different dynamic and static performance.

SVPWM technology is applied to accept the driving voltage in α-β coordinate and output a set of switching instruction to control the 6 switches in full bridge inverter.

Position and speed estimator is designed to observe the real time motor speed through the motor driving voltage and current. The estimated motor speed is compared with the expected speed, and the comparison result serves as the input of the speed PI regulator. The estimated rotor position angle is used by the coordinate transformation unit.

Figure 9. FOC Control Diagram




# 4 Core Modules

## 4.1 Coordinate Transformation

The coordinate transformation includes Clark transformation and Park transformation. On the contrary, the inverse coordinate transformation includes inverse Clark transformation and inverse Park transformation.

1. Clark transformation

   It changes a quantity in A-B-C coordinate to α-β coordinate.

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} x_A \\ x_B \end{bmatrix} \ldots (4.1-1)$$

2. Park transformation

   It changes a quantity in α-β coordinate to d-q coordinate.

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} \ldots (4.1-2)$$

3. Inverse Clark transformation

   It changes a quantity in α-β coordinate to A-B-C coordinate.

$$\begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} \ldots (4.1-3)$$

$$x_C = -x_A - x_B \ldots (4.1-4)$$

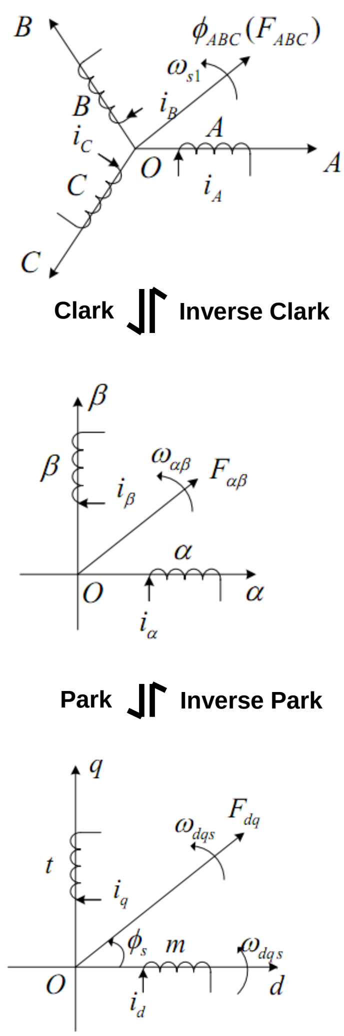
4. Inverse Park transformation

   It changes a quantity in d-q coordinate to α-β coordinate.

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} \ldots (4.1-5)$$

The following figures show the progress of coordinate transformation.

Figure 10. Coordinate Transformation




1. $F_{ABC}, F_{\alpha\beta}, F_{dq}$ -- Synthesis magnetic field
2. $\omega_{s1}, \omega_{\alpha\beta}, \omega_{dqs}$ -- Magnetic field rotating speed
3. $\omega_{s1} = \omega_{\alpha\beta} = \omega_{dqs}$

## 4.2 PI Regulator

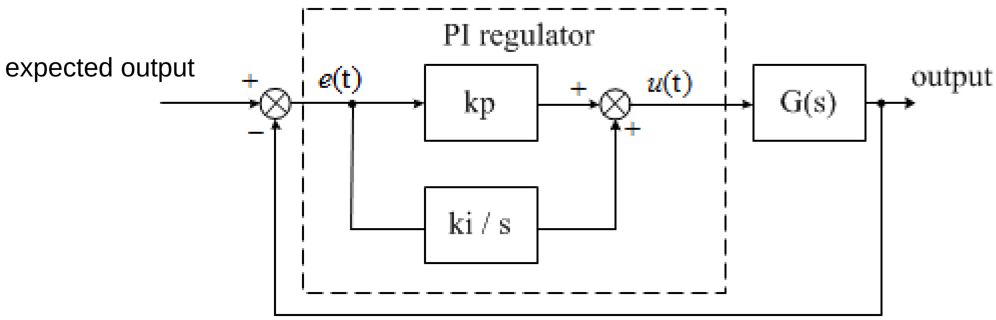
### 4.2.1 Introduction

The PI regulator is composed of a proportional regulator and an international regulator, which mainly has two functions:

1. To assure a fast response when the input is changed;
2. To assure the output follows the given input.

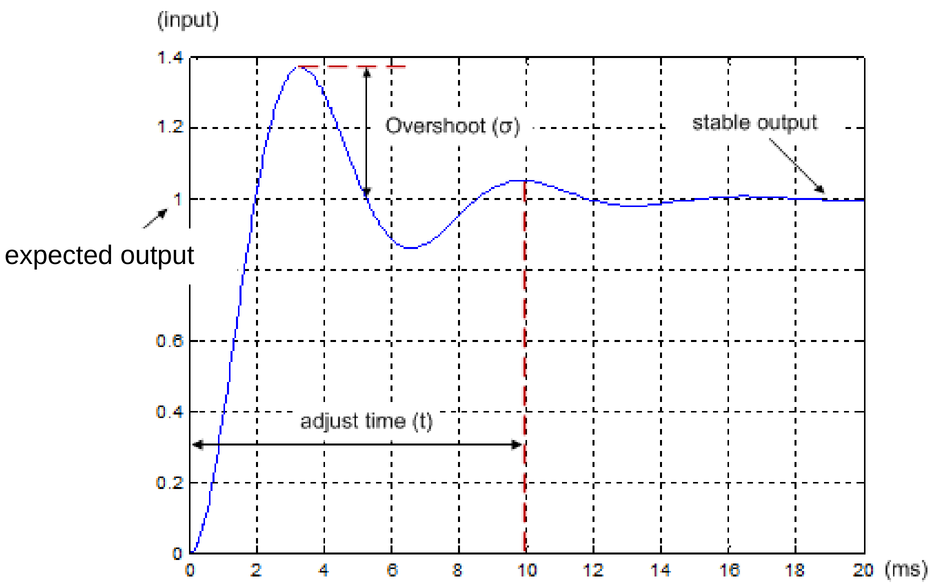
The PI regulator keeps the output follow the expected output by a comparing error between the expected output and the real output. The P-value is to make a fast output response to the comparing error, and the I-value is to decrease the stable output error. Its transfer function can be expressed as follows.

Figure 11. PI-regulator Transfer Function




PI regulator causes a fluctuating output, fluctuating amplitude is decreasing. After some regulating periods, the output follows the expected output with a very small fluctuation around the expected output value.

Figure 12. Output of PI Regulator



### 4.2.2 Formula

The mathematic formula of PI regulator is:

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau \ldots (4.2-1)$$

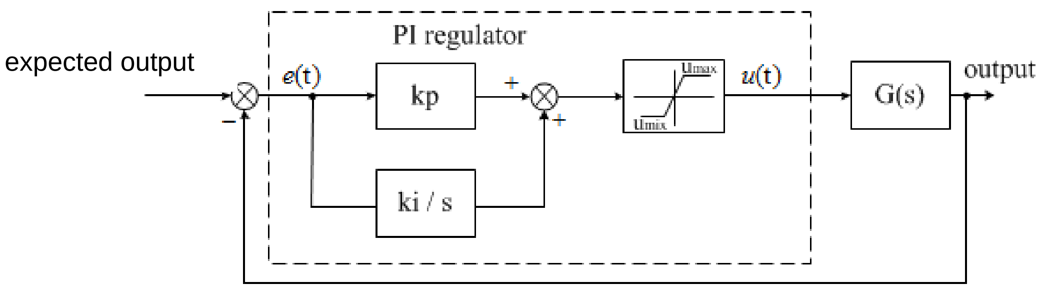
Usually the formula can be changed to the discrete field and expressed by the incremental algorithm.

$$\Delta u(k) = k_p[e(k) - e(k-1)] + k_i e(k) \ldots (4.2-2)$$

$$u(k) = u(k-1) + \Delta u(k) \ldots (4.2-3)$$

Generally, an output limitation is defined to limit the output into a legal range.

Figure 13. PI Regulator with Output Limitation




$$u(k) = u(k-1) + \Delta u(k) \ldots (4.2-4)$$

$$u(k) = u_{max}, if\ u(k) > u_{max} \ldots (4.2-5)$$

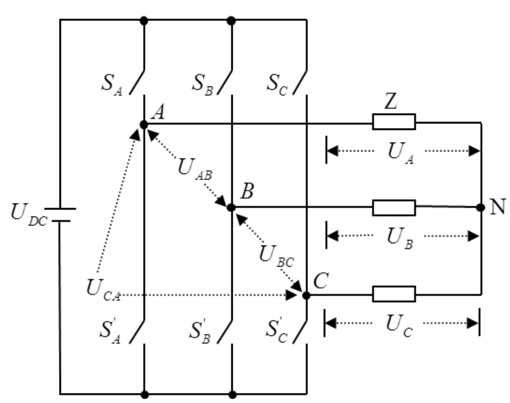
$$u(k) = u_{min}, if\ u(k) < u_{min} \ldots (4.2-6)$$

## 4.3 SVPWM

### 4.3.1 Synthesis of a Space Vector

SVPWM is a method to generate a round rotational voltage vector by controlling the status of 6 switches ($S_A, S_A', S_B, S_B', S_C, S_C'$) in the 3-phase full bridge inverter.

Figure 14. 3-phase Full Bridge Inverter



Below figure lists 8 switching statuses (1 – switch is closed, 0 – switch is opened) and the relative voltages.

Table 1. Eight Switching Statuses and Voltages

| $S_A$ | $S_B$ | $S_C$ | $U_A$ | $U_B$ | $U_C$ | $U_{AB}$ | $U_{BC}$ | $U_{CA}$ | Vector |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0_{000}$ |
| 1 | 0 | 0 | $2U_{DC}/3$ | $-U_{DC}/3$ | $-U_{DC}/3$ | $U_{DC}$ | 0 | $-U_{DC}$ | $U_0$ |
| 1 | 1 | 0 | $U_{DC}/3$ | $U_{DC}/3$ | $-2U_{DC}/3$ | 0 | $U_{DC}$ | $-U_{DC}$ | $U_{60}$ |
| 0 | 1 | 0 | $-U_{DC}/3$ | $2U_{DC}/3$ | $-U_{DC}/3$ | $-U_{DC}$ | $U_{DC}$ | 0 | $U_{120}$ |
| 0 | 1 | 1 | $-2U_{DC}/3$ | $U_{DC}/3$ | $U_{DC}/3$ | $-U_{DC}$ | 0 | $U_{DC}$ | $U_{180}$ |
| 0 | 0 | 1 | $-U_{DC}/3$ | $-U_{DC}/3$ | $2U_{DC}/3$ | 0 | $-U_{DC}$ | $U_{DC}$ | $U_{240}$ |
| 1 | 0 | 1 | $U_{DC}/3$ | $-2U_{DC}/3$ | $U_{DC}/3$ | $U_{DC}$ | $-U_{DC}$ | 0 | $U_{300}$ |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $0_{111}$ |

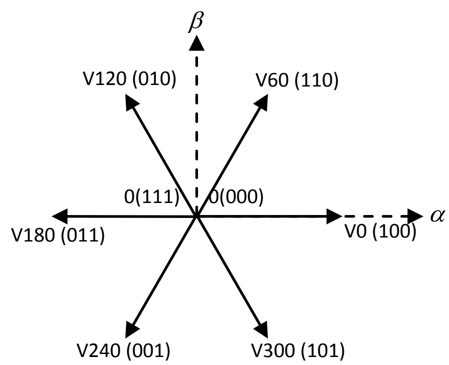
Through the Clark transformation, those voltages are equally converted to α-β coordinate.

Table 2. Voltages in α-β Coordinate

| $S_A$ | $S_B$ | $S_C$ | $U_\alpha$ | $U_\beta$ | Vector |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | $0_{000}$ |
| 1 | 0 | 0 | $2U_{DC}/3$ | 0 | $U_0$ |
| 1 | 1 | 0 | $U_{DC}/3$ | $U_{DC}/\sqrt{3}$ | $U_{60}$ |
| 0 | 1 | 0 | $-U_{DC}/3$ | $U_{DC}/\sqrt{3}$ | $U_{120}$ |
| 0 | 1 | 1 | $-2U_{DC}/3$ | 0 | $U_{180}$ |
| 0 | 0 | 1 | $-U_{DC}/3$ | $-U_{DC}/\sqrt{3}$ | $U_{240}$ |
| 1 | 0 | 1 | $U_{DC}/3$ | $-U_{DC}/\sqrt{3}$ | $U_{300}$ |
| 1 | 1 | 1 | 0 | 0 | $0_{111}$ |

Then 6 basic voltage vectors and 2 zero vectors are got.

Figure 15. Basic Voltage Vectors




Because motor is driven by a magnetic field created by the 3-phase stator winding, let's see the relationship between the voltage and the magnetic field. A winding follows below voltage-flux formula.

$$\vec{u}_s = R_s\vec{\iota}_s + \frac{d\vec{\psi}_s}{dt} \dots (4.3.1-1)$$

For the motor winding, if the motor rotational speed is not very low, the formula above can be simplified as:

$$\vec{v}_s = \frac{d\vec{\psi}_s}{dt} \dots (4.3.1-2)$$

So we can get:

$$\vec{\psi}_s = \int_0^t \vec{v}_s d\tau \dots (4.3.1-3)$$

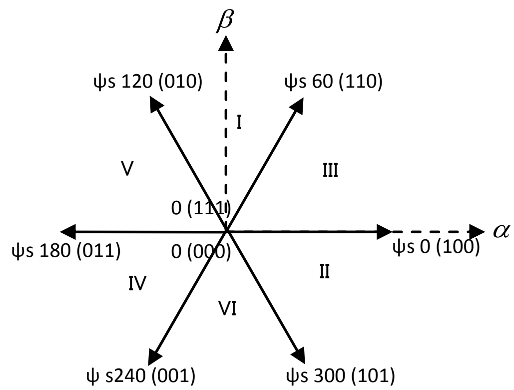
From the table of 'voltages in α-β coordinate', it is easy to know all the basic voltages vectors are constants. So the formula above can become:

$$\vec{\psi}_s = \int_0^T \vec{v}_s dt = \vec{v}_s \int_0^T d\tau = \vec{v}_s \times T \dots (4.3.1-4)$$

It means that the stator magnetic field is created by the voltage added in motor stator winding. Furthermore, a longer time of voltage applying on winding causes a bigger magnetic field.

In Figure 15, 6 basic voltage vectors divide the space into 6 numbers. It is numbered as follows (Voltage vectors are expressed as magnetic vectors).

Figure 16. Sector Numbers



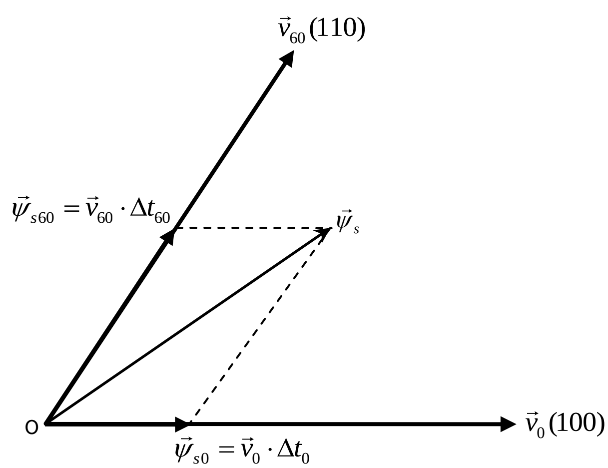

$$\vec{\psi}_k = \vec{V}_k \times T \ (k = 0, 60, 120,180,240,300) \ \ldots (4.3.1-5)$$

In a control period T, if we divide the T into 3 parts: $\Delta T_0(action\ time\ of\ V_0)$, $\Delta T_{60}(action\ time\ of\ V_{60})$. $\Delta T_{null}(the\ zero\ vectors\ action\ time)$, a magnetic field can be expressed as:

$$\vec{\psi}_s = \vec{V}_0 \times \Delta T_0 + \vec{V}_{60} \times \Delta T_{60} = \vec{\psi}_{s0} + \vec{\psi}_{s60} + \vec{\psi}_{null} \ \ldots (4.3.1-6)$$

Below figure shows the synthesis magnetic vector.

Figure 17. Vector Synthesis

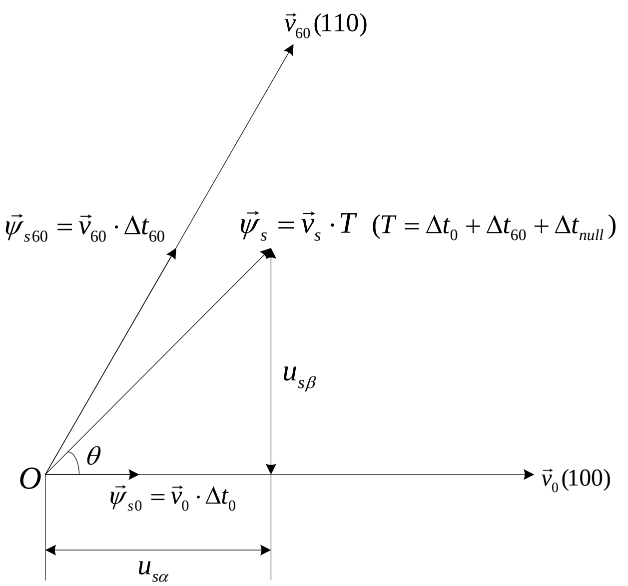



In above figure, it shows the synthesis vector in sector III. By the same way, the synthesis magnetic $\vec{\psi}_s$ of any direction in every sector can be composed of the adjacent 2 basic vectors and the zero vectors.

### 4.3.2 Basic Vector Conduction Time

Assuming the synthesis vector locates in the sector III, the conduction time can be calculated by the following method.

Figure 18. Conduction Time of Basic Vectors




From the table of Eight Switching Statuses and Voltages, It is easy to know the basic vector amplitude is a constant ($|\vec{v}_0| = |\vec{v}_{60}| = \frac{2}{3}V_{DC}$).

The synthesis vector $\vec{\psi}_s$ is composed of $\vec{\psi}_{s0}$ and $\vec{\psi}_{s60}$.

$$\begin{cases} |\vec{v}_s| \cdot T \cdot \cos\theta = |\vec{v}_0| \cdot \Delta t_0 \cdot \cos 0° + |\vec{v}_{60}| \cdot \Delta t_{60} \cdot \cos 60° \\ |\vec{v}_s| \cdot T \cdot \sin\theta = |\vec{v}_0| \cdot \Delta t_0 \cdot \sin 0° + |\vec{v}_{60}| \cdot \Delta t_{60} \cdot \sin 60° \\ |\vec{v}_0| = |\vec{v}_{60}| = \frac{2}{3}V_{DC} \\ u_{s\alpha} = |\vec{v}_s| \cdot T \cdot \cos\theta \\ u_{s\beta} = |\vec{v}_s| \cdot T \cdot \cos\theta \end{cases} \quad ...(4.3.2-1)$$

It can be simplified as

$$\begin{cases} \frac{\Delta t_0}{T} = \frac{\frac{3}{2}u_{s\alpha} - \frac{\sqrt{3}}{2}u_{s\beta}}{V_{DC}} \\ \frac{\Delta t_{60}}{T} = \frac{\sqrt{3}u_{s\beta}}{V_{DC}} \end{cases} \quad ...(4.3.2-2)$$

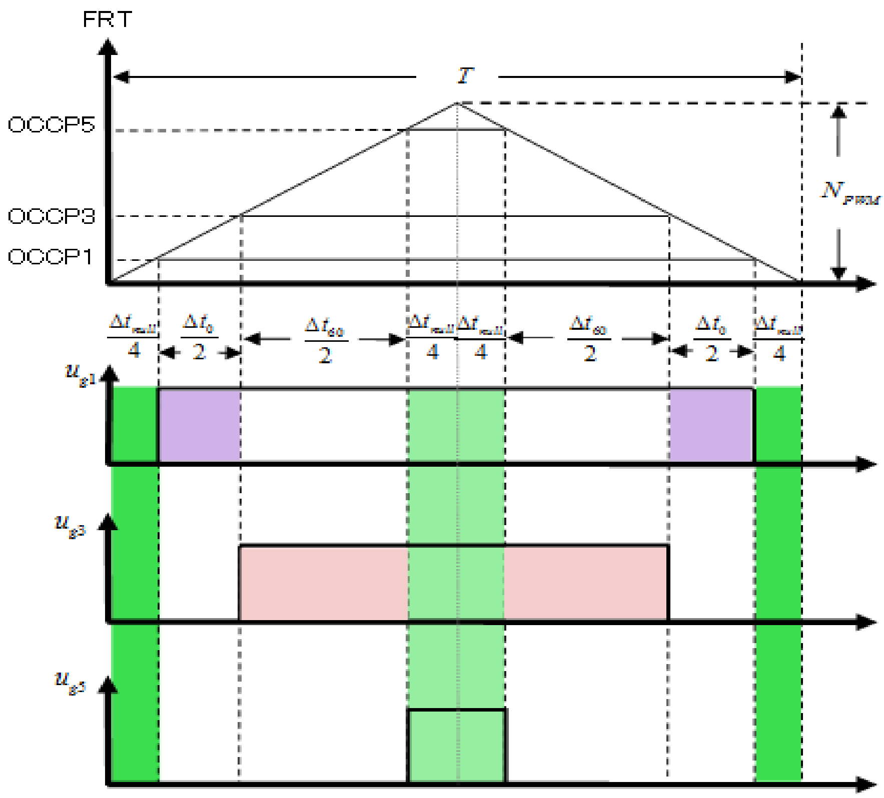
It means that the conduction time of basic vectors can be calculated from $u_{s\alpha}, u_{s\beta} and V_{dc}$.

If $T > \Delta T_{60} + \Delta T_0$, the rest time is filled by the zero vectors.

For other sectors, the conduction time of basic vectors can be got similarly.

The condition time control of basic vectors can be done by the timer in MCU. Generally MCU offers an up-down counter. According to the counting value, we control the full bridge switches to assign the conduction time for each basic vector.

Figure 19. Assign the Conduction Time of Basic Vectors by an Up-down Counter




FRT is an up-down counter in FMx series MCU. Npwm is FRT's counting top value.

$u_{g1}$ controls $S_A$ and $S'_A$. OCCP1 is the $u_{g1}$ toggle time.

$u_{g3}$ controls $S_B$ and $S'_B$. OCCP3 is the $u_{g3}$ toggle time.

$u_{g5}$ controls $S_C$ and $S'_C$. OCCP5 is the $u_{g5}$ toggle time.

### 4.3.3 Sector Number Calculation

According to the sector division, the sector number can be judged by the following table.

Table 3. Sector Number Calculation

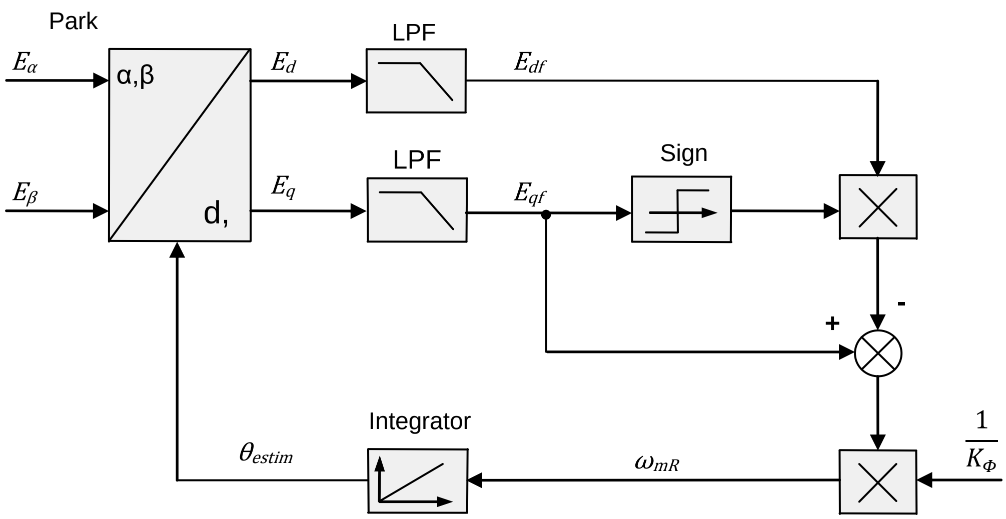
| Sector | Condition |
|---|---|
| I | $v_\beta > 0, \frac{v_\beta}{\lvert v_\alpha \rvert} > \sqrt{3}$ |
| II | $v_\alpha > 0, v_\beta < 0, -\frac{v_\beta}{v_\alpha} < \sqrt{3}$ |
| III | $v_\alpha > 0, v_\beta > 0, \frac{v_\beta}{v_\alpha} < \sqrt{3}$ |
| IV | $v_\alpha < 0, v_\beta < 0, \frac{v_\beta}{v_\alpha} < \sqrt{3}$ |

| Sector | Condition |
|---|---|
| V | $v_\alpha < 0, v_\beta > 0, -\frac{v_\beta}{v_\alpha} < \sqrt{3}$ |
| VI | $v_\beta < 0, -\frac{v_\beta}{\lvert v_\alpha \rvert} > \sqrt{3}$ |

## 4.4 Sensor-less Position Observer

The estimator has PLL structure. Its operating principle is based on the fact that the d-component of the Back Electromotive Force (BEMF) must be equal to zero at a steady state functioning mode. The block diagram of the estimator is presented in below figure.

Figure 20. PLL Estimator's Block Schematic




Starting from the closed loop shown in Figure 4-2, the estimated speed (ωmR) of the rotor is integrated in order to obtain the estimated angle, as shown in Equation 4.4-1:

$$\text{Equation 1: } \theta_{estim=\int \omega_{mR} dt} \ldots (4.4-1)$$

The estimated speed, ωmR, is obtained by dividing the q-component of the BEMF value with the voltage constant, KΦ, as shown in Equation 4.4-2.

$$\text{Equation 2: } \omega_{mR} = \frac{1}{K_\Phi}\left(E_{qf} - \text{sign}\left(E_{qf}\right) \cdot E_{df}\right) \ldots (4.4-2)$$

Considering the initial estimation premise (the d-axis value of BEMF is zero at steady state) shown in Equation 2, the BEMF q-axis value, Eqf, is corrected using the d-axis BEMF value, Edf, depending on its sign. The BEMF d-q component's values are filtered with a first order filter, after their calculation with the Park transform, as indicated in Equation 4.4-3.

$$\text{Equation 3: } \begin{cases} E_d = E_\alpha \cos(\theta_{estim}) + E_\beta \sin(\theta_{estim}) \\ E_q = E_\beta \cos(\theta_{estim}) - E_\alpha \sin(\theta_{estim}) \end{cases} \ldots (4.4-3)$$

With the fixed stator frame, Equation 4.4-4 represents the stators circuit equations.

$$\text{Equation 4: } \begin{cases} E_\alpha = V_\alpha - R_s I_\alpha - L_s \frac{dI_\alpha}{dt} \\ E_\beta = V_\beta - R_s I_\beta - L_s \frac{dI_\beta}{dt} \end{cases} \ldots (4.4-4)$$

In Equation 4, the terms containing α – β were obtained from the three-phase system's corresponding measurements through Clarke transform. Ls and Rs represent the stator inductance and resistance per phase respectively, considering Y (star) connected stator phases. If the motor is Δ (delta) connected, the equivalent Y connection phase resistance and inductance should be calculated and used in the equations above.

# 5 Mathematical Model of a 3-Phase PMSM

FOC is a control theory based on coordinate transformation. In the d-q coordinate, the PMSM mathematical model can be described as below.

$$v_d = R_s i_d + \frac{d\psi_d}{dt} - \omega_r \psi_q \; ... (5-1)$$

$$v_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_r \psi_d \; ... (5-2)$$

$$T_e = \frac{3}{2} n_p (\psi_d i_q - \psi_q i_d) \; ... (5-3)$$

$$\psi_d = L_d i_d + \lambda_m \; ... (5-4)$$

$$\psi_q = L_q i_q \; ... (5-5)$$

Where,

$v_d$ – voltage of d axis
$v_q$ – voltage of q axis
$i_d$ – current of d axis
$i_q$ – current of q axis
$L_d$ – inductance of d axis
$L_q$ – inductance of q axis
$\psi_d$ – magnetic linkage of d axis
$\psi_q$ – magnetic linkage of q axis
$R_s$ – motor stator phase resistor
$\omega_r$ – rotor rotational speed
$\lambda_m$ – permanent magnetic linkage of rotor
$n_p$ – pole pairs

In FOC method, it always keeps $i_d$ to zero so that the motor mathematical model can be simplified in the steady status.

$$v_d = \frac{d\psi_d}{dt} - \omega_r \psi_q \; ... (5-6)$$

$$v_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_r \psi_d \; ... (5-7)$$

$$T_e = \frac{3}{2} n_p \psi_d i_q \; ... (5-8)$$

$$\psi_d = \lambda_m \; ... (5-9)$$

$$\psi_q = L_q i_q \; ... (5-10)$$

From the equation above, the following results can be deduced:

1. The magnetic exciting is independently determined by the permanent magnetic linkage ($\lambda_m$);
2. Torque is independently determined by the current on q axis ($i_q$);

So we can adjust only the current on q axis ($i_q$) to control the torque which determines the motor rotational speed.

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For more Information on Cypress semiconductor products, visit the following website:

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## Document History

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|---|---|---|---|---|
| ** | — | SHEY | 02/26/2015 | Initial release |
| *A | 5232902 | SHEY | 04/21/2016 | Migrated Spansion Application Note from FM3_ AN709-00015-1v0-E to Cypress format |
| *B | 5795967 | AESATMP8 | 07/03/2017 | Updated logo and Copyright. |

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