Electromagnetic Radiation

Relations between Radiated Power, Power-Density, Antenna-Parameters and Received-Power.

Application Note

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1 Introduction

The relations between various parameters described by the theory covering the electromagnetic field are well known. But there are a lot of different formulas for different parameters and depending on different parameters, which could be confusing sometimes. Furthermore sometimes the meaning or exact definition of a parameter is not always clear. So for instance, the r (distance) in the formulas, explained below, is a certain distance, but it’s not always clear which distance is defined by this parameter. Therefore this Application-Note should represent a collection of the various formulas for the various parameters in dependence of different parameters and should furthermore help to clarify what’s the exact meaning of the various parameters.

2 Derivations

The received power P_r (at the Receiver-Antenna) can be calculated from the transmitted power P_t (from the Transmitter-Antenna) and the distance r_r between the Receiver-Antenna and Transmitter-Antenna (and the Antenna-Gain of the Receiver-Antenna g_r and the Transmitter-Antenna g_t):

$$P_r = P_t \cdot g_r \cdot g_t \cdot \frac{\lambda^2}{(4 \cdot \pi \cdot r_r)^2} \quad (1$$

It can also be calculated from the Power-Density $S_{i\,r}$ at the Receiver-Antenna and the Antenna-Effective-Aperture (effective Area) of the Receiver-Antenna A_{er}:

$$P_r = A_{er} \cdot S_{i\,rt} \quad (2$$

The transmitted power of an isotropic Antenna equals the entire power (through the area of a sphere) in a distance r_t of the Transmitter-Antenna and could be calculated from the Power-Density $S_{i\,rt}$, the distance r_t from the Transmitter-Antenna and the size of the surface of the sphere with the radius r_t (which means the distance to the Transmitter-Antenna; see also Figure 1):

$$P_t = S_{i\,rt} \cdot 4 \cdot \pi \cdot r_t^{\,2} \quad (3$$

So of course the radiated power equals the power-density in a certain distance r multiplied with the size of the surface of the sphere with the distance (radius) r around the Transmitter-Antenna (see also Figure 1). Whereas $4.\pi.r^2$ in formula 3 represents the size of the surface of the sphere around the antenna in the respective distance.

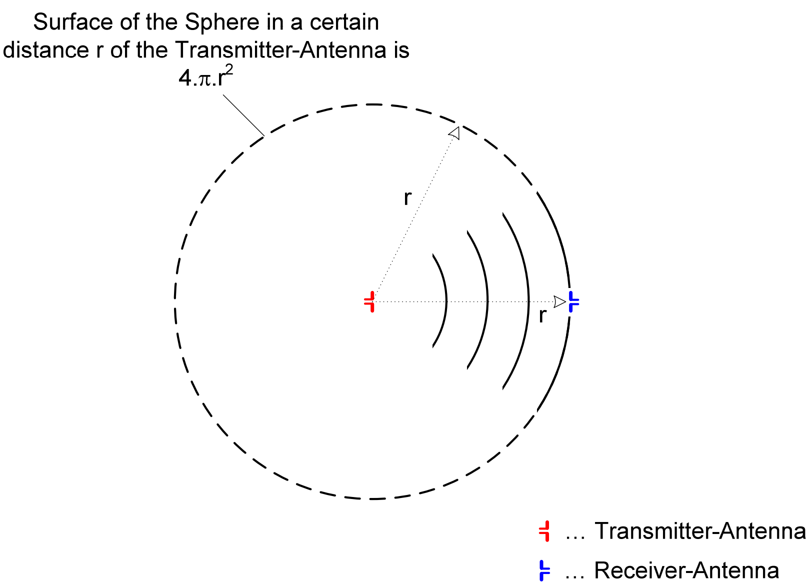

Figure 1 E-Field of an assumed isotropic Transmitter-Antenna

The same could be done for an arbitrary kind or type of antenna (which means also for antenna types different to an isotropic antenna) by correcting the formula 3 by the antenna-gain g_t:

$$P_t = S_{i\,rt} \cdot \frac{4 \cdot \pi \cdot r_t^{\,2}}{g_t} \qquad (4$$

Consequently the received power can be derived from formula 1 and formula 4 as:

$$P_r = g_r \cdot \frac{\lambda^2}{4 \cdot \pi} \cdot S_{i\,rt} \qquad (5$$

So the „Antenna-Effective-Aperture“ of the Receiver-Antenna A_{er} can be defined as:

$$A_{er} = g_r \cdot \frac{\lambda^2}{4 \cdot \pi} \qquad (6$$

And furthermore also the „Antenna-Effective-Aperture“ of the Transmitter-Antenna A_{et} and generally A_e:

$$A_{et} = g_t \cdot \frac{\lambda^2}{4 \cdot \pi} \qquad A_e = g \cdot \frac{\lambda^2}{4 \cdot \pi} \qquad (7, (8$$

As can be seen quite easily, formula (2 can also be derived from formula 5 and formula 6.

Furthermore a formula for the Power-Density $S_{i\,rt}$ in dependence of the radiated power and the distance from the Transmitter-Antenna r_t can derived from formula 4:

$$S_{i\,rt} = g_t \cdot \frac{P_t}{4 \cdot \pi \cdot r_t^{\,2}} \qquad (9$$

The general definition of the Power-Density S_i in dependence of field-strength E is:
(Compare: $P = U^2/R$)

$$S_i = \frac{E^2}{Z_0}$$

$$Z_0 \; in \; free \, air \, (vacuum) \approx Z \; in \, air \; = 120 \cdot \pi$$

(10a, (10b

$$S_i = \frac{E^2}{120 \cdot \pi}$$

Considering the general definition of the Power-Density S_i, the power radiated by the Transmitter-Antenna P_t can be calculated from the field-strength E_{rt} measured in a distance r_t from the Transmitter-Antenna, derived from formula 4 and formula 10a:

$$P_t = \frac{{E_{rt}}^2}{Z_0} \cdot \frac{4 \cdot \pi \cdot {r_t}^2}{g_t} \qquad (11$$

Or in case of an isotropic Antenna (means $g_t = 1$):

$$P_t = \frac{{E_{rt}}^2}{Z_0} \cdot 4 \cdot \pi \cdot {r_t}^2 \qquad (12$$

Now the received power P_r can also be calculated from the electrical field-strength E_{rt} in a distance r_t from the Transmitter-Antenna and the distance r_r from the Receiver-Antenna (and the characteristic free air wave resistance Z_0 and the wavelength λ), derived from formula 1 and formula 11:

$$P_r = g_r \cdot \frac{{E_{rt}}^2}{Z_0} \cdot \frac{\lambda^2 \cdot {r_t}^2}{4 \cdot \pi \cdot {r_r}^2} \qquad (13$$

Or, of course, also from the electrical field-strength E_{rr}, measured directly on the Receiver-Antenna or in a distance equal to the distance between the Transmitter-Antenna and the Receiver-Antenna r_r (means $r_t = r_r$ and is therefore canceled out):

$$P_r = g_r \cdot \frac{{E_{rr}}^2}{Z_0} \cdot \frac{\lambda^2}{4 \cdot \pi} \qquad (14$$

Contrary to this, for instance a formula for the electrical field strength E_{rt} as a function of the radiated power and the distance r_t to the Transmitter-Antenna can be derived from formula 11:

$$E_{rt} = \frac{1}{r_t} \cdot \sqrt{g_t \cdot \frac{P_t \cdot Z_0}{4 \cdot \pi}} \qquad (15$$

Of course also the field strength E_{rr} at the Receiver-Antenna as a function of the received power P_r and as a function of the distance r_r between the Receiver-Antenna and the Transmitter-Antenna can be derived from formula (14:

$$E_{rr} = \frac{1}{\lambda} \cdot \sqrt{4 \cdot \pi \cdot \frac{P_r \cdot Z_0}{g_r}} \qquad (16$$

As given in formula 9, the power-density S_i can be calculated from the radiated power (from the Transmitter-Antenna) and the distance to the Transmitter-Antenna r_t:

$$S_{i\,rt} = \frac{E^2}{Z_0} = g_t \cdot \frac{P_t}{4 \cdot \pi \cdot r_t^{\,2}} \tag{9}$$

But the power density S_i at the Receiver-Antenna or at the same distance as the distance of the Receiver-Antenna r_r can of course also be calculated from the received power P_r (at the Receiver-Antenna), the wave-length and the antenna gain g_r:

$$S_{i\,rr} = \frac{P_r}{A_{er}} = \frac{P_r}{g_r \cdot \lambda^2} \cdot 4 \cdot \pi \tag{17}$$

Furthermore also the voltage at the Receiver-Antenna, in case of a matched antenna, can be calculated (compare $P = U^2/R$):

$$U_r = \sqrt{P_r \cdot Z_0} = \lambda \cdot E_{rr} \sqrt{\frac{g_r}{4\pi}} \tag{18}$$

The so called path attenuation (or path loss), describing the reduction of the received power at a certain (fixed) antenna area, could be easily derived from formula 1, when assuming the antenna gain of the Receiver-Antenna gr and the antenna gain of the Transmitter-Antenna as 1. In this case the path attenuation is the ratio between the radiated power P_t and the received power P_r or 10 times the logarithm of this ratio when defining the path attenuation, as usual, in dB:

$$a_r = 10 \cdot \log\left(\frac{P_t}{P_r}\right) = 20 \cdot \log\left(\frac{4 \cdot \pi \cdot r_r}{\lambda}\right) \tag{19}$$

3 Conclusion

The received power P_r is, of course, proportional to the transmitted power and the effective antenna area. It is proportional to the power-density S_r at the respective distance r_r and the „Antenna-Effective-Aperture" of the Receiver-Antenna A_{er}. Both, the power-density and consequently the received power are inversely proportional to the size of the surface of the sphere around the radiation centre through the receiving point (radius of the sphere is equal to the distance r_r), which is, of course, $4.\pi.r^2$. Furthermore the received power is proportional to wave length, thus it's proportional to the ratio of the wave length and the size of the surface of the respective sphere around the radiation centre. Or in other words, the received power is proportional to the wave length and inversely proportional to the size of the surface of the sphere at the receiving point, which could be understood quite easily, as the radiated power at a certain distance is spread over the entire surface of the respective sphere.

4 Terms and Definitions

Symbol	Unit	Description
P	[W]	Power (generally)
P_t	[W]	radiated Power
P_r	[W]	received Power
r_t	[m]	distance to the Transmitter-Antenna at a certain point
r_r	[m]	distance of the Receiver-Antenna to the Transmitter-Antenna
S_i	[W/m^2]	Power-Density (generally)
$S_{i\,rt}$	[W/m^2]	Power-Density at the distance r_t from the Transmitter-Antenna
$S_{i\,rr}$	[W/m^2]	Power-Density at the Receiver-Antenna or at a point at the distance r_r
E	[V/m]	Electrical Field-Strength (generally)
E_{rt}	[V/m]	Electrical Field-Strength at the distance r_t
E_{rr}	[V/m]	Electrical Field-Strength at the Receiver Antenna or at the distance r_r
g	[1] or [W/W]	Antenna-Gain (generally)
g_r	[1] or [W/W]	Antenna-Gain of the Receiver-Antenna
g_t	[1] or [W/W]	Antenna-Gain of the Transmitter-Antenna
A_e	[m^2]	Antenna-Effective-Aperture (generally)
A_{er}	[m^2]	Antenna-Effective-Aperture of the Receiver-Antenna
A_{et}	[m^2]	Antenna-Effective-Aperture of the Transceiver-Antenna
Z_0	[Ω]	Characteristic Wave Resistance in free Air
λ	[m]	Wave-Length (of the radiated signal; λ=c/f)
a_r	[dB]	Path Attenuation or Path Loss
U_r	[V]	Received voltage in case of matching

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