Electromagnetic Radiation
Relations between Radiated Power, Power-Density, Antenna-Parameters and Received-Power.

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Wireless Control
Electromagnetic Radiation

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1 Introduction

The relations between various parameters described by the theory covering the electromagnetic field are well known. But there are a lot of different formulas for different parameters and depending on different parameters, which could be confusing sometimes. Furthermore sometimes the meaning or exact definition of a parameter is not always clear. So for instance, the $r$ (distance) in the formulas, explained below, is a certain distance, but it's not always clear which distance is defined by this parameter. Therefore this Application-Note should represent a collection of the various formulas for the various parameters in dependence of different parameters and should furthermore help to clarify what's the exact meaning of the various parameters.
2 Derivations

The received power $P_r$ (at the Receiver-Antenna) can be calculated from the transmitted power $P_t$ (from the Transmitter-Antenna) and the distance $r$ between the Receiver-Antenna and Transmitter-Antenna (and the Antenna-Gain of the Receiver-Antenna $g_r$ and the Transmitter-Antenna $g_t$):

$$P_r = P_t \cdot g_r \cdot g_t \cdot \frac{\lambda^2}{(4 \cdot \pi \cdot r_r)^2}$$

(1)

It can also be calculated from the Power-Density $S_{i,t}$ at the Receiver-Antenna and the Antenna-Effective-Aperture (effective Area) of the Receiver-Antenna $A_{er}$:

$$P_r = A_{er} \cdot S_{i,rt}$$

(2)

The transmitted power of an isotropic Antenna equals the entire power (through the area of a sphere) in a distance $r_t$ of the Transmitter-Antenna and could be calculated from the Power-Density $S_{i,n}$, the distance $r_t$ from the Transmitter-Antenna and the size of the surface of the sphere with the radius $r_t$ (which means the distance to the Transmitter-Antenna; see also Figure 1):

$$P_t = S_{i,rt} \cdot 4 \cdot \pi \cdot r_t^2$$

(3)

So of course the radiated power equals the power-density in a certain distance $r$ multiplied with the size of the surface of the sphere with the distance (radius) $r$ around the Transmitter-Antenna (see also Figure 1). Whereas $4 \cdot \pi \cdot r^2$ in formula 3 represents the size of the surface of the sphere around the antenna in the respective distance.

Figure 1 E-Field of an assumed isotropic Transmitter-Antenna
The same could be done for an arbitrary kind or type of antenna (which means also for antenna types different to an isotropic antenna) by correcting the formula 3 by the antenna-gain $g_t$:

$$P_t = S_{i,rt} \cdot \frac{4 \cdot \pi \cdot r_t^2}{g_t} \quad (4)$$

Consequently the received power can be derived from formula 1 and formula 4 as:

$$P_r = g_r \cdot \frac{\lambda^2}{4 \cdot \pi} \cdot S_{i,rt} \quad (5)$$

So the „Antenna-Effective-Aperture“ of the Receiver-Antenna $A_{er}$ can be defined as:

$$A_{er} = g_r \cdot \frac{\lambda^2}{4 \cdot \pi} \quad (6)$$

And furthermore also the „Antenna-Effective-Aperture“ of the Transmitter-Antenna $A_{et}$ and generally $A_e$:

$$A_{et} = g_t \cdot \frac{\lambda^2}{4 \cdot \pi} \quad A_e = g \cdot \frac{\lambda^2}{4 \cdot \pi} \quad (7, (8)$$

As can be seen quite easily, formula (2) can also be derived from formula 5 and formula 6.

Furthermore a formula for the Power-Density $S_{i,rt}$ in dependence of the radiated power and the distance from the Transmitter-Antenna $r_t$ can derived from formula 4:

$$S_{i,rt} = g_t \cdot \frac{P_t}{4 \cdot \pi \cdot r_t^2} \quad (9)$$

The general definition of the Power-Density $S_i$ in dependence of field-strength $E$ is:

(Compare: $P = U^2/R$)

$$S_i = \frac{E^2}{Z_0} \quad (10a, (10b)$$

$Z_0$ in free air (vacuum) $\approx Z$ in air $= 120 \cdot \pi$
Electromagnetic Radiation

Considering the general definition of the Power-Density $S_0$, the power radiated by the Transmitter-Antenna $P_t$ can be calculated from the field-strength $E_{rt}$ measured in a distance $r_t$ from the Transmitter-Antenna, derived from formula 4 and formula 10a:

$$P_t = \frac{E_{rt}^2}{Z_0} \cdot \frac{4 \cdot \pi \cdot r_t^2}{g_t}$$  \hspace{1cm} (11)

Or in case of an isotropic Antenna (means $g_t = 1$):

$$P_t = \frac{E_{rt}^2}{Z_0} \cdot 4 \cdot \pi \cdot r_t^2$$  \hspace{1cm} (12)

Now the received power $P_r$ can also be calculated from the electrical field-strength $E_{rt}$ in a distance $r_t$ from the Transmitter-Antenna and the distance $r_r$ from the Receiver-Antenna (and the characteristic free air wave resistance $Z_0$ and the wavelength $\lambda$), derived from formula 1 and formula 11:

$$P_r = g_r \cdot \frac{E_{rt}^2}{Z_0} \cdot \frac{\lambda^2 \cdot r_t^2}{4 \cdot \pi \cdot r_r^2}$$  \hspace{1cm} (13)

Or, of course, also from the electrical field-strength $E_{rr}$, measured directly on the Receiver-Antenna or in a distance equal to the distance between the Transmitter-Antenna and the Receiver-Antenna $r_r$ (means $r_t = r_r$ and is therefore canceled out):

$$P_r = g_r \cdot \frac{E_{rr}^2}{Z_0} \cdot \frac{\lambda^2}{4 \cdot \pi}$$  \hspace{1cm} (14)

Contrary to this, for instance a formula for the electrical field strength $E_{rt}$ as a function of the radiated power and the distance $r_t$ to the Transmitter-Antenna can be derived from formula 11:

$$E_{rt} = \frac{1}{r_t} \cdot \sqrt{g_t \cdot \frac{P_t \cdot Z_0}{4 \cdot \pi}}$$  \hspace{1cm} (15)

Of course also the field strength $E_{rr}$ at the Receiver-Antenna as a function of the received power $P_r$ and as a function of the distance $r_r$ between the Receiver-Antenna and the Transmitter-Antenna can be derived from formula (14):

$$E_{rr} = \frac{1}{\lambda} \cdot \sqrt{4 \cdot \pi \cdot \frac{P_r \cdot Z_0}{g_r}}$$  \hspace{1cm} (16)
As given in formula 9, the power-density $S_i$ can be calculated from the radiated power (from the Transmitter-Antenna) and the distance to the Transmitter-Antenna $r_t$:

$$S_{i,rt} = \frac{E^2}{Z_0} = g_t \cdot \frac{P_t}{4 \cdot \pi \cdot r_t^2} \quad (9)$$

But the power density $S_i$ at the Receiver-Antenna or at the same distance as the distance of the Receiver-Antenna $r_r$ can of course also be calculated from the received power $P_r$ (at the Receiver-Antenna), the wavelength and the antenna gain $g_r$:

$$S_{i,rr} = \frac{P_r}{A_{er}} = \frac{P_r}{g_r \cdot \lambda^2} \cdot 4 \cdot \pi \quad (17)$$

Furthermore also the voltage at the Receiver-Antenna, in case of a matched antenna, can be calculated (compare $P = U^2/R$):

$$U_r = \sqrt{P_r \cdot Z_0} = \lambda \cdot E_{rr} \sqrt{\frac{g_r}{4\pi}} \quad (18)$$

The so called path attenuation (or path loss), describing the reduction of the received power at a certain (fixed) antenna area, could be easily derived from formula 1, when assuming the antenna gain of the Receiver-Antenna $g_r$ and the antenna gain of the Transmitter-Antenna as 1. In this case the path attenuation is the ratio between the radiated power $P_t$ and the received power $P_r$ or 10 times the logarithm of this ratio when defining the path attenuation, as usual, in dB:

$$a_r = 10 \cdot \log \left( \frac{P_t}{P_r} \right) = 20 \cdot \log \left( \frac{4 \cdot \pi \cdot r_r}{\lambda} \right) \quad (19)$$
3 Conclusion

The received power $P_r$ is, of course, proportional to the transmitted power and the effective antenna area. It is proportional to the power-density $S_r$ at the respective distance $r$, and the „Antenna-Effective-Aperture“ of the Receiver-Antenna $A_{\text{eff}}$. Both, the power-density and consequently the received power are inversely proportional to the size of the surface of the sphere around the radiation centre through the receiving point (radius of the sphere is equal to the distance $r$), which is, of course, $4.\pi r^2$. Furthermore the received power is proportional to wave length, thus it’s proportional to the ratio of the wave length and the size of the surface of the respective sphere around the radiation centre. Or in other words, the received power is proportional to the wave length and inversely proportional to the size of the surface of the sphere at the receiving point, which could be understood quite easily, as the radiated power at a certain distance is spread over the entire surface of the respective sphere.
## 4 Terms and Definitions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>P</td>
<td>[W]</td>
<td>Power (generally)</td>
</tr>
<tr>
<td>Pt</td>
<td>[W]</td>
<td>radiated Power</td>
</tr>
<tr>
<td>Pr</td>
<td>[W]</td>
<td>received Power</td>
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<tr>
<td>rt</td>
<td>[m]</td>
<td>distance to the Transmitter-Antenna at a certain point</td>
</tr>
<tr>
<td>rr</td>
<td>[m]</td>
<td>distance of the Receiver-Antenna to the Transmitter-Antenna</td>
</tr>
<tr>
<td>Si</td>
<td>[W/m²]</td>
<td>Power-Density (generally)</td>
</tr>
<tr>
<td>Si rt</td>
<td>[W/m²]</td>
<td>Power-Density at the distance rt from the Transmitter-Antenna</td>
</tr>
<tr>
<td>Si rr</td>
<td>[W/m²]</td>
<td>Power-Density at the Receiver-Antenna or at a point at the distance rr</td>
</tr>
<tr>
<td>E</td>
<td>[V/m]</td>
<td>Electrical Field-Strength (generally)</td>
</tr>
<tr>
<td>E rt</td>
<td>[V/m]</td>
<td>Electrical Field-Strength at the distance rt</td>
</tr>
<tr>
<td>E rr</td>
<td>[V/m]</td>
<td>Electrical Field-Strength at the Receiver Antenna or at the distance rr</td>
</tr>
<tr>
<td>g</td>
<td>[1] or [W/W]</td>
<td>Antenna-Gain (generally)</td>
</tr>
<tr>
<td>Ae</td>
<td>[m²]</td>
<td>Antenna-Effective-Aperture (generally)</td>
</tr>
<tr>
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<td>[m²]</td>
<td>Antenna-Effective-Aperture of the Receiver-Antenna</td>
</tr>
<tr>
<td>Aret</td>
<td>[m²]</td>
<td>Antenna-Effective-Aperture of the Transceiver-Antenna</td>
</tr>
<tr>
<td>Z0</td>
<td>[Ω]</td>
<td>Characteristic Wave Resistance in free Air</td>
</tr>
<tr>
<td>λ</td>
<td>[m]</td>
<td>Wave-Length (of the radiated signal; λ=c/f)</td>
</tr>
<tr>
<td>ar</td>
<td>[dB]</td>
<td>Path Attenuation or Path Loss</td>
</tr>
<tr>
<td>Ur</td>
<td>[V]</td>
<td>Received voltage in case of matching</td>
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